A certified module of Simplicial Complexes for the Kenzo system*

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1 Scope of the work

In the field of *Intelligent Information Processing*, mechanized reasoning systems provide a chance of increasing the reliability of software systems, namely *Computer Algebra Systems*. This paper is devoted to a concrete case of this topic.

The notion of simplicial complex, see [7], is the most elementary method to settle a connection between common "general" topology and homological algebra. The notion of topological space is too "abstract" in order to perform computations. A triangulation, by means of simplicial complexes, can be provided for "sensible" spaces, so every topological space can be considered as a simplicial complex, making the computations easier.

Nevertheless, many common constructions in topology are difficult to make explicit in the framework of simplicial complexes. It soon became clear in the forties the notion of simplicial set is much better. The reference [7] remains the basic reference in this subject.

The Kenzo system [2] is a Common Lisp program which works with the main mathematical structures used in Simplicial Algebraic Topology, namely it is able to work with simplicial sets. However the notion of simplicial complex is not included in the Kenzo system.

Kenzo was written mainly as a research tool and has got relevant results which have not been confirmed nor refuted by any other means. Then, the question of Kenzo reliability (beyond testing) arose in a natural way. Several works (see [1] and [6]) have focussed on studying the correctness of first order *fragments* of Kenzo with the ACL2 theorem prover [5].

We have undertaken two tasks: on the one hand, the development of a new Kenzo module which integrates the notion of simplicial complex. On the other hand, certifying the correctness of this module using the ACL2 theorem prover. The complete development can be found in [3].

2 A new certified Kenzo module

The new Kenzo module consists of two main programs. From a set of maximal elements, a simplicial complex can be generated [7]. The first program,

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called `simplicial-complex-generator`, implements this functionality. The second program, called `ss-from-sc`, constructs from a simplicial complex the simplicial set canonically associated (see [7]). This new module is fully integrated in the Kenzo system, hence, all the Kenzo functionalities can be employed in the new constructions, for instance, the homology groups of a simplicial set canonically associated to a simplicial complex can be computed.

Our second goal consists in proving the correctness of the two programs previously presented by means of the ACL2 theorem prover.

The formalization of `simplicial-complex-generator` in ACL2 is split in two steps. First of all, the necessary definitions related to simplicial complexes are introduced in ACL2. Subsequently, theorems stating that from a set of maximal elements the program *really* constructs the associated simplicial complex are proven.

Finally, the certification of the correctness of `ss-from-sc`, that constructs from a simplicial complex the simplicial set canonically associated, is provided by means of the Generic Simplicial Set theory tool [4]. This tool reduces the proving effort for each family of simplicial sets, letting ACL2 automates the routine parts of the proof. In this way, the fact that from a simplicial complex the program *really* constructs a simplicial set is proven.

3 Conclusions and further work

The module presented in this paper allows one to work with simplicial complexes, a sensible representation for topological spaces. The implementation has been written in Common Lisp, enhancing the Kenzo system but also allowing us to certify the correctness of the programs in ACL2. The development of new certified Kenzo modules remains as further work.

References

1. M. Andrés, L. Lambán, J. Rubio, and J. L. Ruiz-Reina. Formalizing Simplicial Topology in ACL2. *Proceedings of ACL2 Workshop 2007*, pages 34–39, 2007.
2. X. Dousson, J. Rubio, F. Sergeraert, and Y. Siret. The Kenzo program. Institut Fourier, Grenoble, 1998. `http://www-fourier.ujf-grenoble.fr/~sergerar/Kenzo/`.
3. J. Heras. ACL2 verification of Simplicial Complexes programs for the Kenzo system. University of La Rioja, 2010. `http://www.unirioja.es/cu/joheras/simplicial-complexes.html`.
4. J. Heras, V. Pascual, and J. Rubio. Proving with ACL2 the correctness of simplicial sets in the Kenzo system. *To appear in Proceedings of 20th International Symposium on Logic-Based Program Synthesis and Transformation*, 2010.
5. M. Kaufmann, P. Manolios, and J. S. Moore. *Computer-Aided Reasoning: An approach*. Kluwer Academic Publishers, 2000.
6. F. J. Martín-Mateos, J. Rubio, and J. L. Ruiz-Reina. ACL2 verification of simplicial degeneracy programs in the Kenzo system. *Lectures Notes in Computer Science*, 5625:106–121, 2009.
7. J. P. May. *Simplicial objects in Algebraic Topology*, volume 11 of *Van Nostrand Mathematical Studies*. 1967.